Gas Pipeline Calculator

1. Introduction

The PPI Gas Pipeline Calculator was developed by the Energy Piping Systems Division for estimating parameters involved in gas flow in plastic pipe. Although this calculator was primarily developed for plastic pipe, it will work for any pipe as long as the correct pipe roughness and internal diameter are entered. This document describes how to use the calculator and explains the variables and equations behind the calculations. This document is broken up in to 5 sections:

- Section 1 is the introduction to the calculator
- Section 2 describes how to use the calculator. Screen shots of the calculator fields are shown and explained.
- Section 3 explains the parameters used in the gas calculator. Users should reference this section for explanation of the parameters used in this calculator.
- Section 4 shows the derivation of the General Flow Equation.
- Section 5 describes many methods for estimating the friction factor and lists reasons why the 5 methods used by this calculator were chosen.

2. Overview of the Calculator

The PPI Gas Pipeline Flow Calculator uses the General Flow Equation to solve for one of the five variables below after the user enters four of these variables and information about the gas mixture. **Figure 2.1** shows a schematic of the 5 pipeline gas flow variables.

1. Gas Flow Rate, Q
2. Pipe Internal Diameter, D_i
3. Pipe Length, L
4. Inlet Pressure, P_1
5. Outlet Pressure, P_2

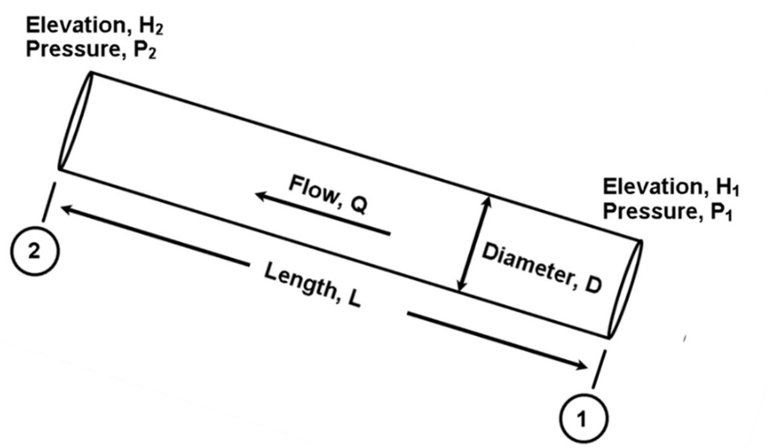

Figure 2.1 – Pipeline Variables

Results are calculated using 5 different methods of estimating frictional resistance so the user can get a feel for the variability in the results due to the friction factor. The General Flow Equation and the

5 different methods of estimating frictional resistance are shown on the Equations Tab (see **Figure 2.2**).

Input Variables | Result | Equations

The PPI Gas Pipeline Flow Calculator uses the General Flow Equation -

$$Q = \frac{C_1}{\sqrt{f}} * \left(\frac{T_b}{P_b}\right) * \left[\frac{(P_1^2 - P_2^2 - H_c)}{(SG * T_a * L * Z_a)}\right]^{0.5} * D^{2.5} * E$$

where,

$$H_c = C_2 * SG * (H_2 - H_1) * P_{avg}^2 / (Z_a * T_a)$$

$$C_1 = 77.58$$

$$C_2 = 0.0375$$

where,

Q = Flow Rate [SCFD]
F = Transmission Factor
Tb = Base Temperature [°R]
Ta = Average Temperature [°R]
Pb = Base Pressure [psia]
P1 = Pressure at Inlet of pipe [psia]
P2 = Pressure at Outlet of pipe [psia]
D = Pipe Inside Diameter [inch]
L = Pipe Length [miles]
SG = Specific Gravity of Gas
Za = Average Gas Compressibility Factor
Pavg = Average Flow Pressure [psia]
E = Pipeline Efficiency
H1 = Elevation at Point 1 [feet]
H2 = Elevation at Point 2 [feet]

Friction Factor, f, can be estimated using following relations -

Colebrook-White (Modified)

$$\frac{1}{\sqrt{f}} = -2 * Log_{10}\left[\frac{\epsilon}{3.7} + \frac{2.825}{(Re * \sqrt{f})}\right]$$

IGT Improved

$$\frac{1}{\sqrt{f}} = 2.3095 * Re^{0.1}$$

Chen

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon}{3.7065} - \frac{5.0452}{Re} * \log_{10} C\right)$$

where,

$$C = \frac{\epsilon^{1.1096}}{2.8257} + \frac{7.149}{Re^{0.8961}}$$

Gouder - Sonnad

$$\frac{1}{\sqrt{f}} = 0.8686 * \ln\left[\frac{0.4587 * Re}{C - 0.31^{C/(C+1)}}\right]$$

where,

$$C = 0.124 * Re * \epsilon + \ln(0.4587 * Re)$$

Renouard

$$\frac{1}{\sqrt{f}} = 0.21 * Re^{-0.2}, Re < 4000$$

$$\frac{1}{\sqrt{f}} = 2.4112 * Re^{0.09}, 4000 < Re < 4e6$$

$$\frac{1}{\sqrt{f}} = 2.1822 * Re^{0.1}, Re > 4e6$$

Figure 2.2 – Equations Tab

The Input Variable for the PPI Gas Pipeline Calculator consists of Pipe Data, Gas Composition, Gas Properties, and Units for the input variables and calculated values. **Input data will remain for each consecutive calculation unless it is manually changed. There isn't a button to clear all fields.** **Figure 2.3** below shows the Pipe Data subtab of the Input Variables tab. The "Solve For" field, the "Pipe Internal Diameter" field, and the "Pipe Roughness" field have pull down menus so you can select a value. The Pipeline Data Fields will change depending on which parameter you select to solve for. Solve for choices are shown in **Figure 2.4**. If you select internal diameter, you need to enter values for Q, L, P1 and P2. If you select P2, you need to enter values for Q, D, L, and P1.

The pull down menu of the Pipe Internal Diameter field can be used to select a pipe size and DR to enter an internal diameter or value can be manually entered. The same is true for the Pipe Roughness field; a value can be selected from the menu or it can be manually entered.

Figure 2.3 – Pipe Data Fields subtab of Input Variables tab

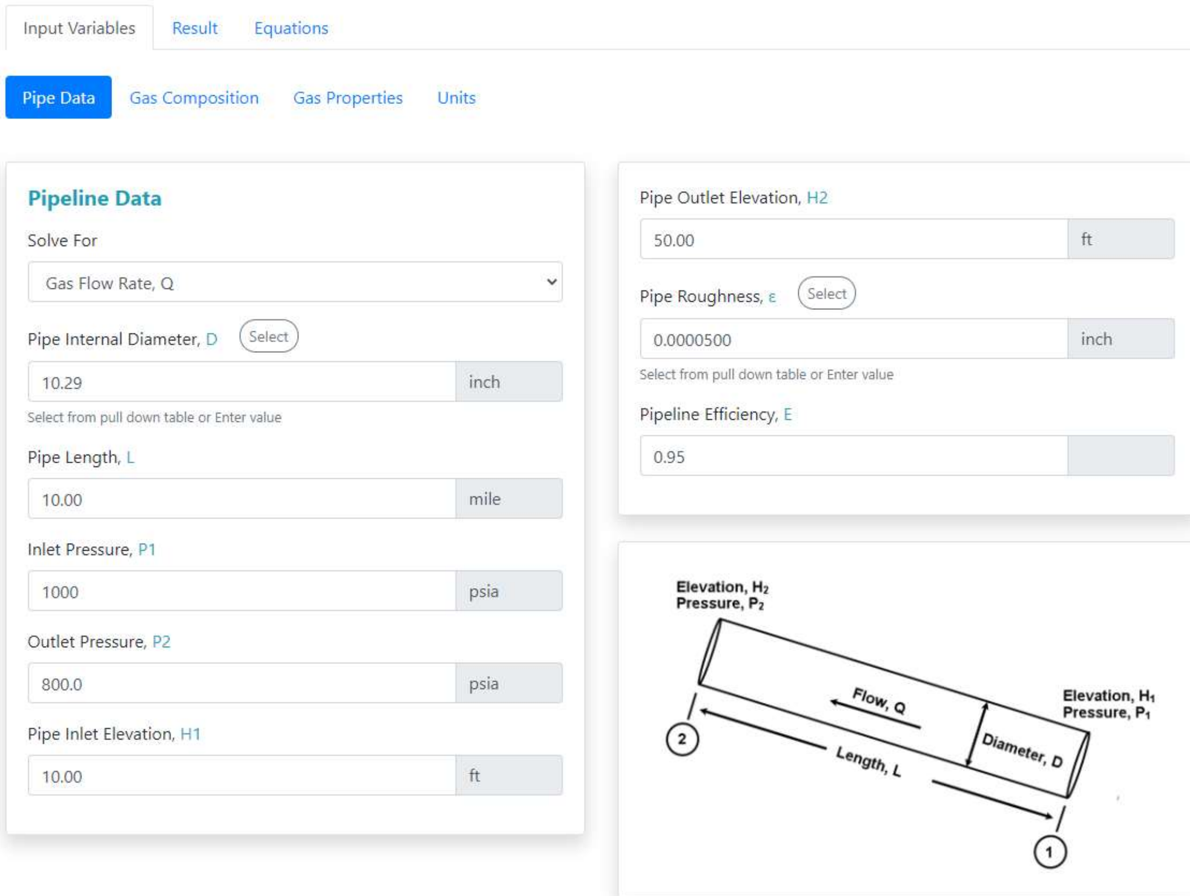

Figure 2.4 – Options to Solve For on pull down menu

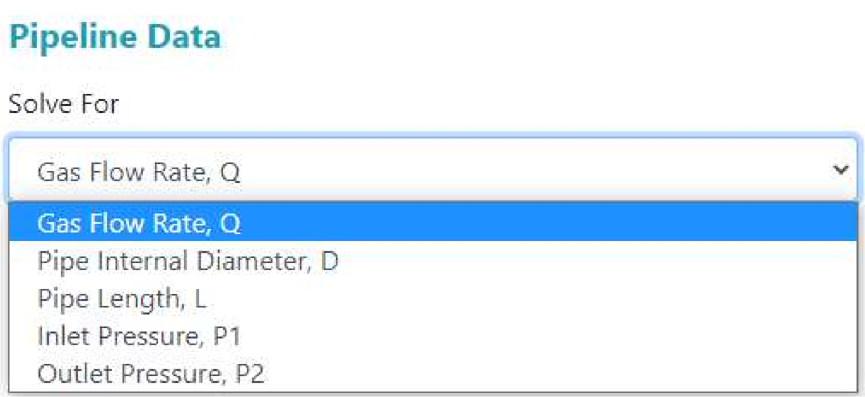

Figure 2.5 below shows the Gas Composition subtab of the Input Variable tab. The composition of the gas can be changed by entering the percentage of each gas in the mixture. Fields at the bottom of the table allow the user to enter variables for a gas that may not be in the table. The percentages of all the gases must equal 100% at the bottom of the table. **Input data will remain for each consecutive calculation unless it is manually changed.** The gas mixture specific gravity, dynamic viscosity, and density are calculated from these percentages. The average compressibility of the gas mixture is determined from the average gas pressure and temperature.

Figure 2.6 below shows the Gas Properties subtab of the Input Variable tab. The default base pressure and base temperature are 14.7psia and 60°F. The user can manually change these values by checking the box next to the variable and entering a different value. The average compressibility factor (z_{avg}) and dynamic viscosity (μ) of the gas mixture will automatically be calculated after the gas composition is entered. These parameters can be manually changed by selecting "User Defined" as the variable determination factor. The methods used to calculate z_{avg} and μ are discussed in Section 3. Other parameters on this Gas Properties subtab that are automatically calculated after entering the gas composition include molecular weight, pseudo-critical temperature, pseudo-critical pressure, specific gravity (G), and gas density (ρ). The inlet and outlet temperatures should be adjusted as needed. If they are unknown, enter the average ground temperature for each.

Figure 2.7 shows the Units subtab of the Input Variables tab. The calculator will convert input variables and calculated values automatically if a variable unit of measure is changed. The units can be changed on the following variables:

- Temperature (°F, °C, K, °R)
- Length (miles, km, m, ft)
- Roughness (inch or mm)
- Elevation (inch, feet, mm, m)
- Velocity (ft/s or m/s)
- Pressure (gauge or absolute: psi, kg/cm^3, bar, atm, kPa)
- Dynamic Viscosity (centipoise, $lb_m/ft/s$, lb_f*s/ft^2, kg/m/s, etc.)

The results are automatically updated anytime a variable is changed. **Figure 2.8** shows the Results tab. A report can be generated by selecting the Report button below the results table. After selecting the Report button, enter project and user data (see **Figure 2.9**) before opening or downloading the report. The report summarizes all input and calculated variables (see **Figure 2.10**).

Gas Pipeline Calculator

Figure 2.5 – Gas Composition subtab of Input Variables

Pipe Data | Gas Composition | Gas Properties | Units

Gas	Formula	MW	Specific Gravity, G	Tc	Pc	Specific Heat Ratio, Cp/Cv	% Gas Mixture
				°F	psia		
Air		28.96	1.00	-220.9	549.1	1.40	50
Ammonia	NH3	17.03	0.5880	270.4	1636	1.32	0.0
Argon	Ar	39.95	1.38	-188.2	706.9	1.66	0.0
Carbon Dioxide	CO2	44.01	1.52	87.89	1070	1.28	0.0
Carbon Monoxide	CO	28.02	0.9673	-220.5	507.0	1.40	0.0
Ethane	C2H6	30.07	1.04	90.05	708.3	1.18	0.0
Ethylene	C2H4	28.05	0.9686	48.65	730.4	1.24	0.0
Helium	He	4.00	0.1382	-450.3	32.33	1.66	0.0
Heptane	C7H16	100.2	3.46	512.7	396.8	1.05	0.0
Hexane	C6H14	86.18	2.98	453.6	430.6	1.06	0.0
Hydrogen	H2	2.02	0.06960	-399.9	188.1	1.41	0.0
Hydrogen Sulfide	H2S	34.08	1.18	212.1	1296	1.32	0.0
i-Butane	iC4H10	58.12	2.01	274.9	529.1	1.19	0.0
i-Pentane	iC5H12	72.15	2.49	369.1	490.8	1.08	0.0
Methane	CH4	16.04	0.5539	-116.6	667.2	1.32	50
n-Butane	nC4H10	58.12	2.01	305.7	551.1	1.18	0.0
n-Pentane	nC5H12	72.15	2.49	385.6	489.4	1.08	0.0
Nitrogen	N2	28.01	0.9671	-232.5	492.3	1.40	0.0
Octane	C8H18	114.2	3.94	564.2	360.1	1.05	0.0
Oxygen	O2	32.00	1.10	-181.4	731.9	1.40	0.0
Propane	C3H8	44.10	1.52	206.0	615.8	1.13	0.0
							0.0
							0.0
						Total (% Gas)	**100.0**

1. Critical temperatures, critical pressures, and specific heat ratios from https://www.engineeringtoolbox.com/specific-heat-ratio-d_608.html

Plastics Pipe Institute Gas Pipeline Calculator

Figure 2.6 – Gas Properties subtab of Input Variables

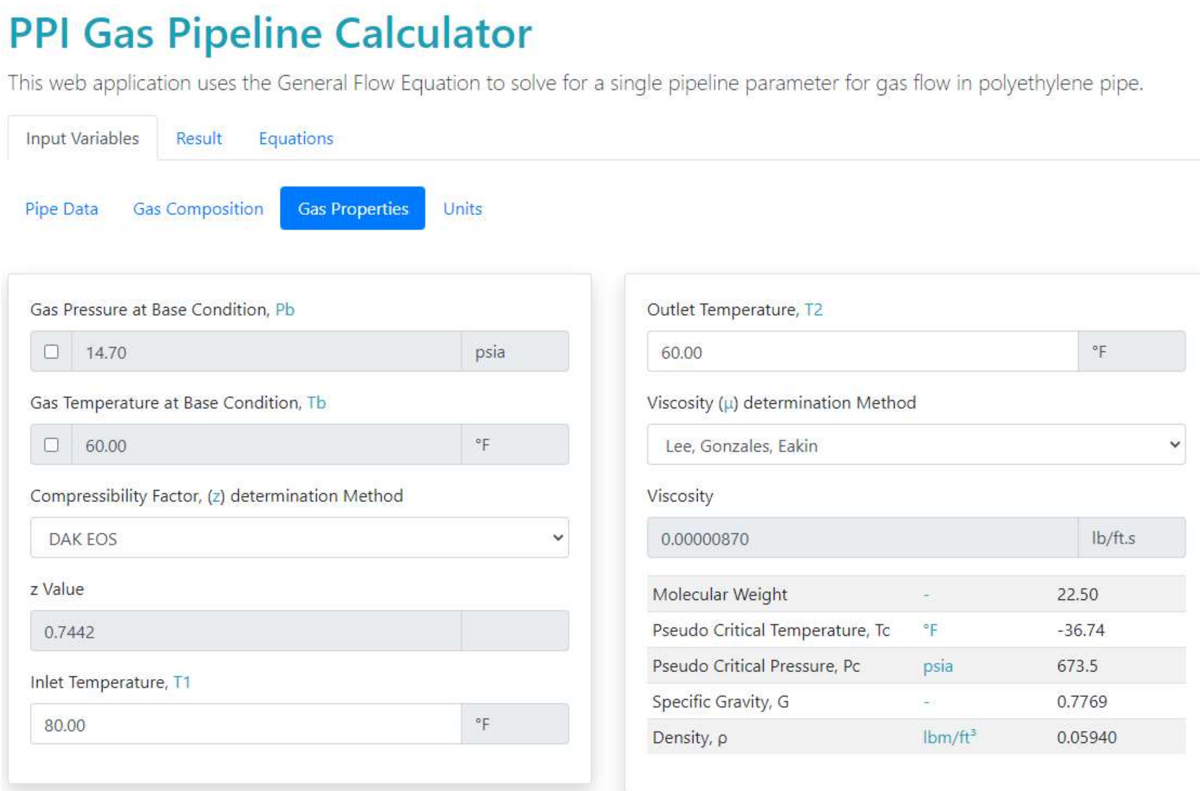

Figure 2.7– Units of Measure subtab of Input Variables

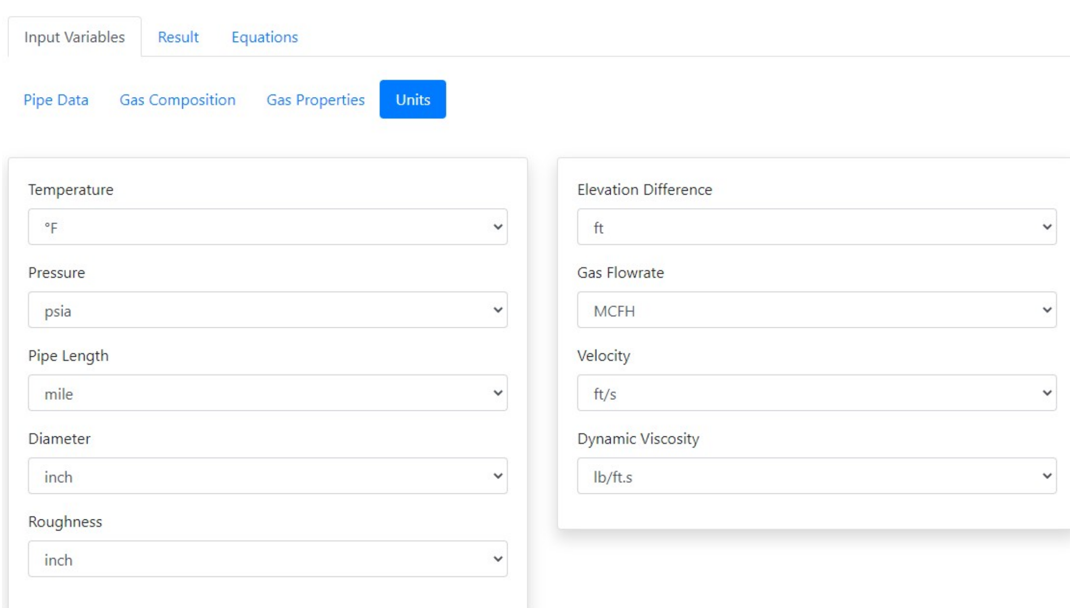

Figure 2.8– Results Tab

Input Variables | Result | Equations

Solution/ Design Parameter based on Transmission Factor Calculation Method

Method		Colebrook-White (Modified)	IGT Improved	Chen	Goudar-Sonnad	Renouard - High
Flowrate	MCFH	4,380	4,906	4,402	4,405	4,607
Friction Factor, f		0.00827	0.00660	0.00819	0.00818	0.00748
Transmission Factor, F		10.99	12.31	11.05	11.06	11.56
Reynold's No., Re		1.66e+7	1.86e+7	1.67e+7	1.67e+7	1.74e+7
Applicable Re Range		4e3 - 1e8	1.6e3 - 3e6	4e3 - 4e8		> 4e6
Pressure, Pavg	psia	903.7	903.7	903.7	903.7	903.7
z Value		0.7442	0.7442	0.7442	0.7442	0.7442
Viscosity, µ	lb/ft.s	0.00000870	0.00000870	0.00000870	0.00000870	0.00000870
Velocity Inlet	ft/s	23.49	26.31	23.61	23.62	24.70
Velocity Outlet	ft/s	29.36	32.89	29.51	29.53	30.88
Erosional Velocity	ft/s	43.33	43.33	43.33	43.33	43.33
Sonic Velocity	ft/s	1088	1088	1088	1088	1088
Mach No.		0.02159	0.02418	0.02170	0.02171	0.02270

Report

Figure 2.9– Report Input Data

Report Options

×

Project Name: Project Name

Developed By: Developer

Date: mm/dd/yyyy

Approved By: Approver

Revision: 0

Reviewed By: Reviewer

Open | Download

Close

Figure 2.10– Report Generated by Calculator

PPI Plastics Pipe Institute ENERGY PIPING SYSTEMS

Energy Piping Systems Division

Gas Pipeline Calculator

https://plasticpipe.org/energy/

Project	*ABC Project*		Developed By	*PPI*
Date	*2/9/2021, 12:12:24 PM*		Reviewed By	
Revision	*0*		Approved By	

This web application uses the General Flow Equation to solve for a single pipeline parameter for gas flow in polyethylene pipe.

Input Data

Calculation For		Gas Flow Rate, Q
Compressibility Factor		DAK EOS
Viscosity		Lee, Gonzales, Eakin
Pipe Inlet Diameter, D	inch	10.29
Pipe Length, L	mile	10.00
Inlet Pressure, P1	psia	1000
Outlet Pressure, P2	psia	800.0

Gas Data

Specific Gravity, G		0.7769
Gas Base Density, ρb	lbm/ft³	0.05940
Average Temperature, Ta	°F	70.00
Compressibility Factor, Z		0.7442
Gas Viscosity, μ	lb/ft.s	0.00000870
Gas Pressure at Base Condition, Pb	psia	14.70
Gas Temperature at Base Condition, Tb	°F	60.00
Pseudo Critical Pressure, Ppr	psia	673.5
Pseudo Critical Temperature, Tpr	°F	-36.74

Pipeline Data

Pipe Roughness, ε	inch	0.0000500
Pipe Inlet Elevation, H1	ft	10.00
Pipe Outlet Elevation, H2	ft	50.00
Pipeline Efficiency, E		0.9500

Solution/ Design Parameter based on Transmission Factor Calculation Method

Method		Colebrook-White (Modified)	IGT Improved	Chen	Goudar-Sonnad	Renouard - High
Flowrate	MCFH	4,380	4,906	4,402	4,405	4,607
Friction Factor, f		0.00827	0.00660	0.00819	0.00818	0.00748
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Applicable Re Range		4e3 - 1e8	1.6e3 - 3e6	4e3 - 4e8		> 4e6
Pressure, Pavg	psia	903.7	903.7	903.7	903.7	903.7
z Value		0.7442	0.7442	0.7442	0.7442	0.7442
Viscosity, μ	lb/ft.s	0.00000870	0.00000870	0.00000870	0.00000870	0.00000870
Velocity Inlet	ft/s	23.49	26.31	23.61	23.62	24.70
Velocity Outlet	ft/s	29.36	32.89	29.51	29.53	30.88
Erosional Velocity	ft/s	43.33	43.33	43.33	43.33	43.33
Sonic Velocity	ft/s	1088	1088	1088	1088	1088
Mach No.		0.02159	0.02418	0.02170	0.02171	0.02270

3.0 Parameters Used in Calculator

This section describes the variables used in calculator and the equations and correlations used to determine variables that are not entered.

Density (ρ) is gas mass per volume. Gas density increases with increasing pressure or decreasing temperature and, decreases with decreasing pressure or increasing temperature. For relatively short lengths of pipe with small pressure drop, the density change will be minimal. In this situation the gas can be treated as incompressible and the Darcy-Weisbach equation can be used. For long pipelines with large pressure differences from inlet to outlet, the density will change appreciably, and a compressible flow equation must be used.

$$\rho = m/V = M*P/(z*R_u*T) \quad [lb_m/ft^3, \text{ slugs}/ft^3, \text{ or } kg/m^3]$$

Specific Weight (γ) is the weight per unit volume, typically expressed in lb_f/ft^3 or kN/m^3. It is equal to the density times the acceleration of gravity (g = 32.17 ft/sec^2 or 9.81 m/s^2).

$$\gamma_{gas} = (\rho_{gas})(g) \left[\frac{lb_m}{ft^3} * 32.2\frac{ft}{s^2}\right]$$

Specific Volume (ν) is the inverse of density. Typical units include: ft^3/slug, or m^3/kg.

$$\nu_{gas} = 1/\rho_{gas} \quad [ft^3/lb_m]$$

Specific Gravity (SG) of a gas is a dimensionless quantity representing the ratio of the density of the gas to the density of air at the same temperature and pressure. The density of air at 60°F & 1 atmosphere (14.7psi) is 0.002373 slugs/ft^3 = 0.002373 lb_f*s^2/ft^4. Air density will change based on temperature, pressure, and humidity.

$$SG = \rho_{gas} / \rho_{air} = M_{gas} / M_{air} = M_{gas} / 28.987 \sim M_{gas} / 29$$

The SG of a gas mixture can be determined using the following equation:

$$SG_{mix} = M_{mix} / M_{air} = \frac{\sum_{i=1}^{n} \%gas_i * M_i}{29}$$

Molecular Weight (M) of a gas is mass per mole. For a gas mixture, it is the summation of the individual molecular components. The molecular weight and specific gravity of a natural gas sample are related to each other by the equation:

$$M_{gas} = (SG_{gas})(M_{air}) \quad [lb_m/lb_{mole}, \text{ g/mole, or } kg/kg_{mole}]$$

where: M_{air} ~ 29 lb_m/lb_{mole}

Compressibility Factor (z) - The compressibility factor or z-factor of a gas is a measure of its deviation from ideal gas law. It is the ratio of the volume occupied by a given amount of gas to the volume occupied by the same amount of ideal gas. At temperatures much greater than the critical temperature of a gas and/or pressures much less than the critical pressure of a gas, it will follow the ideal gas law and the compressibility factor will be one. If the temperature is low enough and/or the pressure is high enough so that the gas will not exhibit ideal gas behavior, then the value of the compressibility factor will be less than one. Excluding compressibility factor or assuming it to be unity results in lower flow rates, higher pressure drops, or larger pipes sizes than would result if compressibility were considered.

The ideal gas law modified with the inclusion of the compressibility factor to account for non-ideal gases is as follows:

$$\mathbf{PV_{ideal}=nR_uT}$$

Substituting z,

$$\mathbf{PV_{actual} = znR_uT}$$

Where:

$z = V_{actual} / V_{ideal}$

P = absolute gas pressure [psia]

V = gas volume [ft^3]

n = **m/M** = gas mass [lb_m] divided by its molecular weight [lb_{moles}]

$\mathbf{R_u}$ = universal gas constant = $10.731 \frac{psia*ft^3}{lb_{mol}*oR} = 1545.4 \frac{ft*lb_f}{lb_{mol}*oR}$

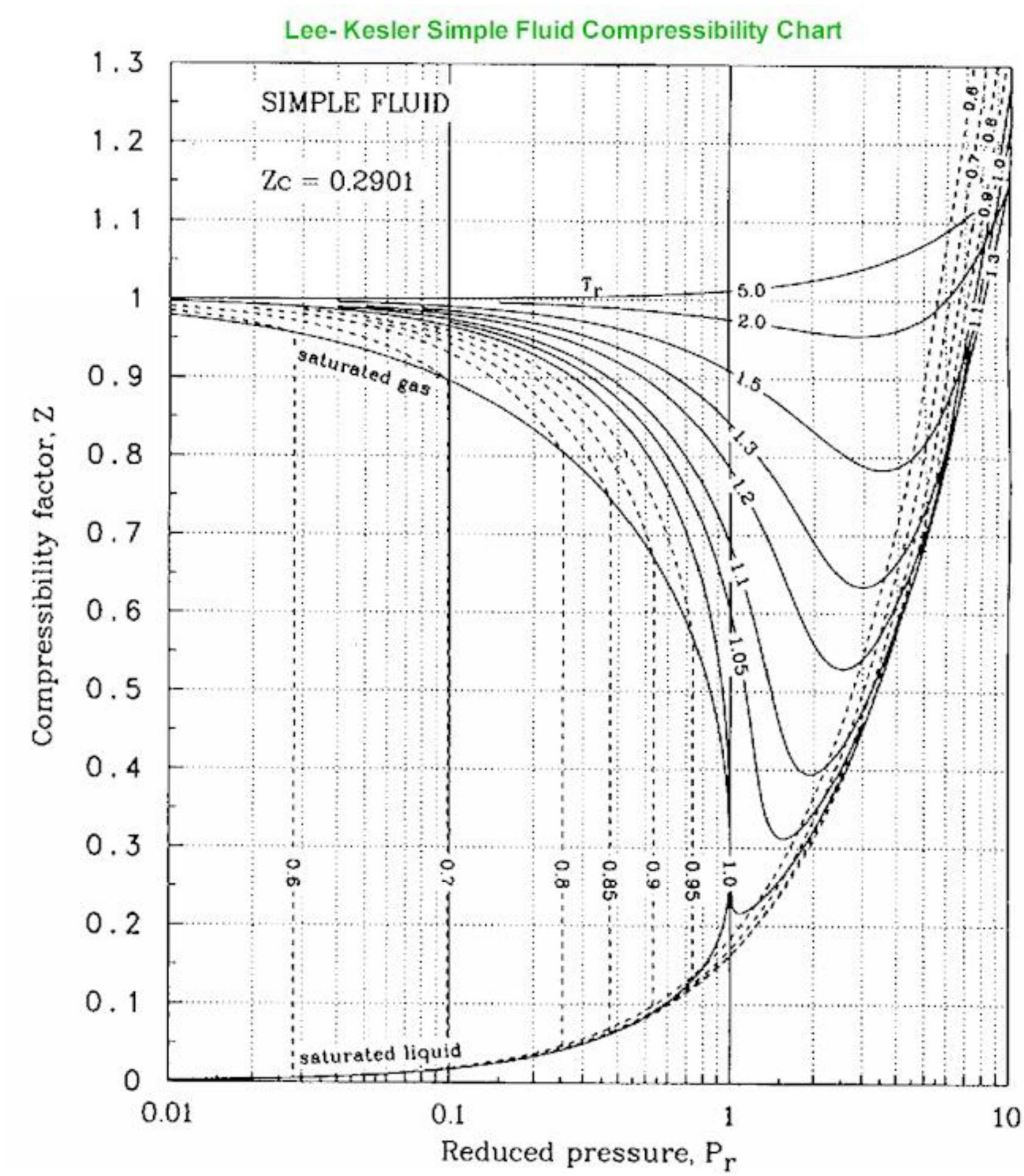
T = gas absolute temperature [°R]

Substituting **n = m/M** and **ρ = m/V** into the ideal gas law equation,

$$\rho = \mathbf{m/V = PM/zR_uT = P*SG*M_{air}/zR_uT}$$

There are several correlations and equations for compressibility factor as a function of temperature and pressure for gases. Correlations depend on the reduced temperature (T_r) and reduced pressure (P_r). T_r and P_r are calculated by dividing the temperature and pressure of the gas by its critical temperature ($T_r=T/T_c$) and critical pressure ($P_r=P/P_c$). These values are then used to find the corresponding z-value on a Lee-Kesler chart (see **Figure 3.1**).

Figure 3.1 – Lee-Kesler Simple Fluid Compressibility Chart



The PPI Gas Calculator estimates the compressibility factor for natural gas based on Dranchuk and Abou-Kassem equation of state (ref). It is expressed as follows:

$$z_{avg} = \left(1 + A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5}\right)\rho_r + \left(A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right)\rho_r^2 - A_9\left(\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right)\rho_r^5 + A_{10} * (1 + A_{11} * \rho_r^2)\frac{\rho_r^2}{T_{pr}^3}e^{(-A_{11}\rho_r^2)}$$

Where

$\rho_r = reduced\ density = \frac{0.27 * P_{pr}}{z * T_{pr}}$

$P_{pr} = psuedo\ reduced\ pressure\ \frac{P\ [psia]}{P_{pc}}$

$T_{pr} = psuedo\ reduced\ temperature = \frac{T\ [oR]}{T_{pc}}$

$P_{pc} =\ pseudo\ crtical\ pressure = \left(4.6 + 0.1 * SG_g - 0.258 * SG_g^2\right) * 10.1325 * 14.7$

$T_{pc} = pseudo\ critical\ temperature = \left(99.3 + 180 * SG_g - 6.94 * SG_g^2\right) * 1.8$

$A_1 = 0.3265$
A¬ = –1.0700
$A_3 = -0.5339$
$A_4 = 0.01569$
$A_5 = -0.05165$
$A_6 = 0.5475$
$A_7 = -0.7361$
$A_8 = 0.1844$
$A_9 = 0.1056$
$A_{10} = 0.6134$
$A_{11} = 0.7210$

Because the parameter z is embedded in ρ_r, an iterative solution is necessary to solve the equation. This estimate has an average absolute error of 0.486% with a standard deviation of 0.00747 over ranges of pseudoreduced pressure and temperature of:

$$0.2 < p_{pr} < 30 \text{ with } 1.0 < T_{pr} < 3.0$$

and

$$p_{pr} < 1.0 \text{ with } 0.7 < T_{pr} < 1.0$$

This equation is not recommended outside these ranges of critical temperature ($T_{pr} \sim 1.0$) and pressures ($p_{pr} > 1.0$).

Dynamic (Absolute) Viscosity (μ) is a quantity measuring the shear force needed to overcome resistance to deformation from internal friction in a fluid or gas. The dynamic viscosity of a fluid is temperature dependent. Newtons Law for shear stress (T) incorporates dynamic viscosity and is defined as:

$$\mathbf{T = F/A = \mu * \mathit{du/dy}} \quad [lb_f/ft^2, N/m^2]$$

Where: F = force [lb_f, N]

A = surface area [ft^2, m^2]

du = change in velocity [ft/s, m/s]

dy = distance between fluid layers [ft, m] *du/dy* = shear rate [s^{-1}]

The PPI Gas Calculator estimates the dynamic viscosity of natural gas using the Lee, Gonzalez and Eakin method.

$$\mu_g = 10^{-4} * k_v * e^{\left[x_v\left(\frac{\rho}{62.4}\right)^{y_v}\right]}$$

Where

$$k_v = \frac{(9.4 + 0.02 * M) * T_{avg}^{1.5}}{\left(209 + 19 * M + T_{avg}\right)}$$

$$y_v = 2.4 - 0.2 * x_v$$

$$x_v = 3.5 + \frac{986}{T} + 0.01 * M$$

T_{avg} = average gas temperature [°R]
ρ = gas density [lb_m/ft^3]
M = molecular weight of the gas [lb_m/lb_{mol}]
μ_g = dynamic viscosity of the gas [cp]

1 centipoise (cp) = 0.0000208854 lb_f*s/ft^2 = 0.000671969 lb_m/ft/s

Kinematic Viscosity (ν) is derived from the ratio of a fluid's dynamic viscosity and its specific weight (density x acceleration of gravity). Two fluids with the same dynamic viscosity can have very different kinematic viscosities depending on their densities. Kinematic Viscosity is expressed as:

$$\nu = \mu / (\rho g) = \mu/\gamma \quad [ft^2/sec,\ m^2/s,\ \text{or centistokes}]$$

Reynolds Number (Re) is a dimensionless quantity used to determine the flow regime (laminar or turbulent) of a moving fluid or gas. For flow in pipes, it is defined as:

$$Re = \frac{vD_i\rho}{\mu} \left[\frac{ft}{s}\frac{ft}{1}\frac{lb_m}{ft^3}\frac{s*ft}{lb_m}\right]$$

Substituting Q/A for v and $\frac{M_{air}SG*P_b}{z*R_u*T_b}$ for ρ,

$$Re = \frac{QD\rho}{A\mu} = \frac{4QD}{\pi D^2\mu} * \frac{M_{air}SG*P_b}{z*R_u*T_b} = \frac{\boldsymbol{Q*SG*P_b}}{\boldsymbol{z*\mu*D*T_b}} * \frac{4*29\frac{lb_m}{lb_{mol}}*\frac{1hr}{3600}*\frac{1000cf}{1Mcf}}{\pi*10.731\frac{psia*ft^3}{lb_{mol}*oR}*\frac{1ft}{12in}}$$

Simplified,

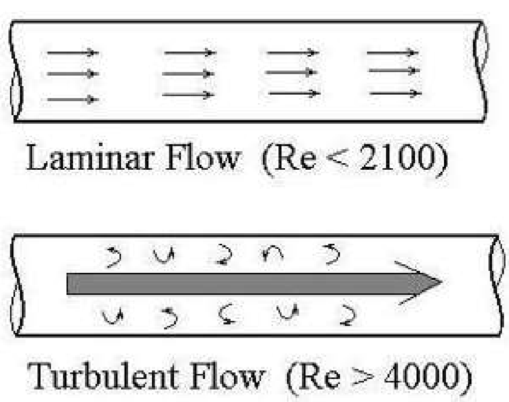
$$Re = 11.46955 * \frac{Q*SG*P_b}{z_{avg}*D_i*\mu*T_b} \left[\frac{ft^3}{s} * \frac{lb_m}{lb_{mol}} * \frac{psia}{ft} * \frac{s*ft}{lb_m} * \frac{1}{oR} * \frac{lb_{mol}*oR}{ft^3*psia}\right]$$

Where:

v = velocity **= Q/A**
Q = flow rate [Mcfh]
A = pipe internal flow area = $\pi \frac{D_i^2}{4}$
D_i = average internal diameter of the pipe [in] = D_o – 2.12*D_o / DR for PE pipe
Where
D_o = outside diameter of pipe
DR = pipe dimension ratio = D_o / t_{min}
t_{min} = minimum PE pipe wall thickness = D_o / DR
μ = dynamic viscosity [lb_m / (ft*s)]
R_u = universal gas constant = 10.731 $\frac{psia*ft^3}{lb_{mol}*oR}$
P_b = base pressure [psia]
T_b = base temperature [°R]

Laminar flow occurs with high viscous fluids traveling at low velocity. In laminar flow, the velocity vectors line up in the direction of flow. Laminar pipe flow occurs at a Re < 2100. Turbulent flow is characterized by mixing with velocity vectors going in all directions, but the overall flow is in one direction. Turbulent flow takes place with low viscous fluids at high velocity. Transport of natural gas in a pipeline is typically turbulent flow. Turbulent flow occurs at Re > 4000. **Figure 3.2** illustrates the velocity vector differences between laminar and turbulent flow in a pipe.

Figure 3.2 – Laminar and Turbulent Flow Velocity Vectors



In the transition region between Reynolds numbers of 2100 and 4000, the flow may be either laminar or turbulent, depending upon factors like the entrance conditions into the pipe and the roughness of the pipe surface.

Roughness Factor (ϵ) is the mean protruding height of relatively uniformly distributed and sized, tightly packed sand grains that would give the same pressure-gradient behavior as the actual protrusions, indentations, and micro-fissures of the pipe wall. Typical units for roughness include inches, feet, and mm. Pipe wall surface roughness is a function of the pipe material, coating type, and pipe age. Corrosion, erosion, and scale buildup over time will increase the roughness factor, reducing flow rates and increasing pressure losses for fluids passing through them. **Figure 3.3** illustrates interior roughness of a pipe. **Table 3.1** lists roughness value ranges for several pipe types.

Figure 3.3 – Pipe wall Interior Roughness

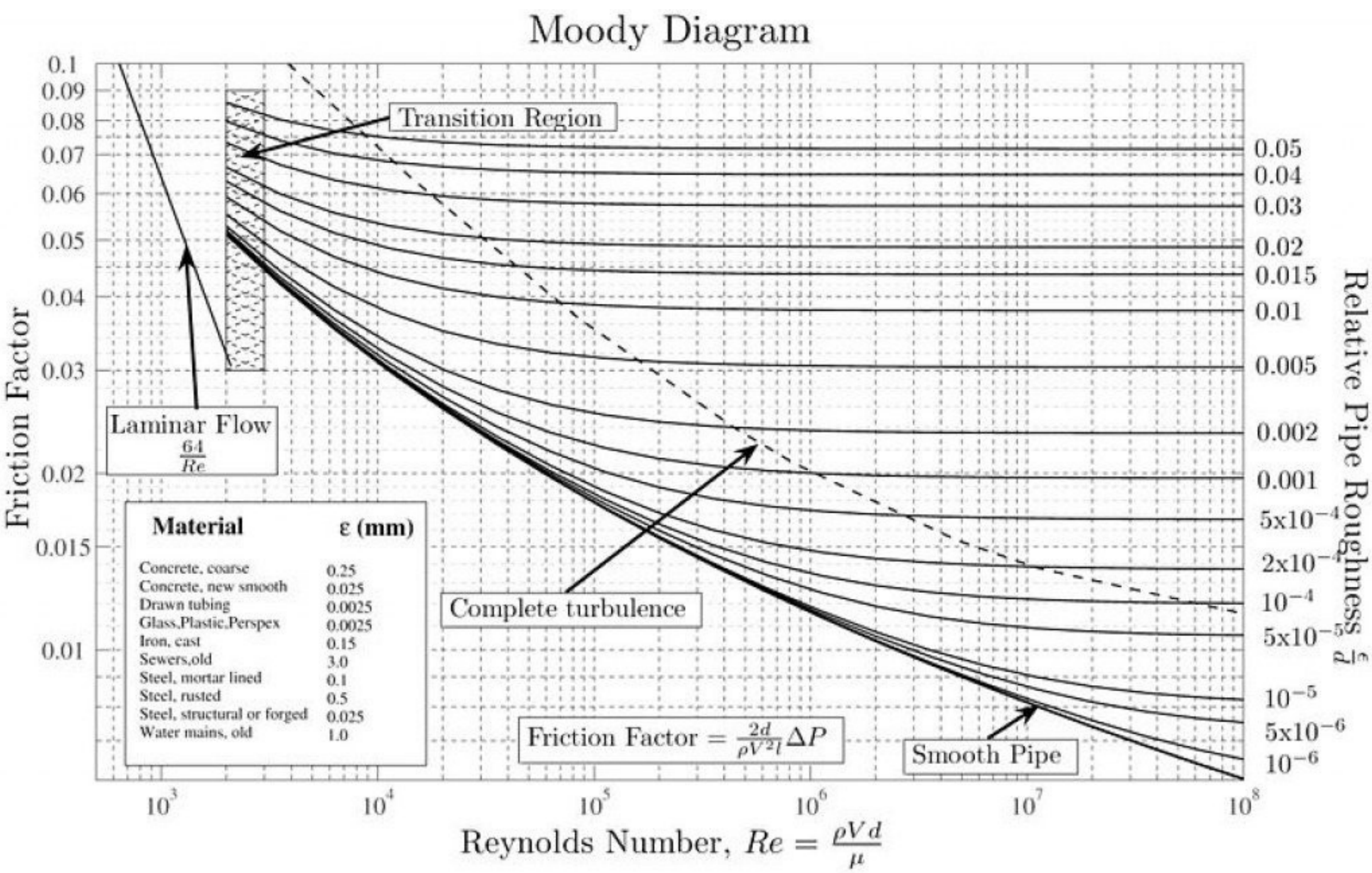
Table 3.1 – Typical Pipe Roughness Values

Pipe Material	Surface roughness, e (ft)	Surface roughness, e (in)
Plastics, HDPE, glass	3 - 7 x 10^{-6}	3.6 - 8.4 x 10^{-5}
Steel, smooth to Welded and lined	30 - 300 x 10^{-6}	3.6 - 36 x 10^{-4}
New Cast/Wrought/Ductile Iron, plain to lined	30 - 800 x 10^{-6}	3.6 - 96 x 10^{-4}
Wood Stave	600 - 3000 x 10^{-6}	72 - 360 x 10^{-4}
Concrete (depends on forming & finish)	1,000 - 16,700 x 10^{-6}	120 – 2000 x 10^{-4}
Cast/Wrought/Ductile Iron, plain, General Tuberculation	2,700 - 8,300 x 10^{-6}	324 – 996 x 10^{-4}
Cast/Wrought/Ductile Iron, plain, Severe Tuberculation & Incrustation	8,300 - 30,000 x 10^{-6}	996 – 3600 x 10^{-4}

Relative Roughness (ϵ =e/D) is the absolute roughness divided by the pipe inside diameter.

Moody friction factor (*f*) is needed for any calculations with the Darcy-Weisbach or General Flow equations. One method of obtaining a value for ***f*** is graphically using the Moody friction factor diagram (see **Figure 3.4**) and values of Re and relative roughness (e/D). Equations for estimating ***f*** are discussed in section 5.

Figure 3.4 – Moody Diagram



Velocity (v) of gas flow represents the speed at which the gas moves in the pipeline. It depends on pressure and will vary along the pipeline due to frictional losses. As the pressure changes the density of the gas also changes. The highest velocity will be at the downstream end of the pipe where the pressure is the least. The lowest velocity will be the upstream end of the pipe where the pressure is higher. Because the mass flow through the pipe is constant, the following relationships can be written:

$$Q_1\rho_1 = Q_2\rho_2 = Q_b\rho_b$$

Therefore,

$Q_1 = Q_b\left(\frac{\rho_b}{\rho_1}\right)$ and $Q_2 = Q_b\left(\frac{\rho_b}{\rho_2}\right)$

Substituting for ρ,

$$Q_1 = Q_b\left(\frac{P_b/_{z_b R T_b}}{P_1/_{z_1 R T_1}}\right) = Q_b \frac{P_b}{P_1}\frac{T_1}{T_b}\frac{z_1}{z_b} \text{ and } Q_2 = Q_b \frac{P_b}{P_2}\frac{T_2}{T_b}\frac{z_2}{z_b}$$

Since Q=v*A and z_b~1,

$$v_1 = \frac{4}{\pi}\frac{Q_b}{D_i^2}\frac{P_b}{P_1}\frac{T_1}{T_b}z_1 = 0.002122\frac{Q_b}{D_i^2}\frac{P_b}{P_1}\frac{T_1}{T_b}z_1$$

and

$$v_2 = \frac{4}{\pi}\frac{Q_b}{D_i^2}\frac{P_b}{P_2}\frac{T_2}{T_b}z_2 = 0.002122\frac{Q_b}{D_i^2}\frac{P_b}{P_2}\frac{T_2}{T_b}z_2$$

Where
Q_b = gas flow rate at standard conditions [ft^3/day]
D_i = pipe internal diameter [inches]
T = temperature [^oR]
P = absolute pressure [psia]

Erosional Velocity (v_{max}) is maximum allowable gas velocity in a pipeline to limit noise and vibration. Acceptable velocities are generally less than 50% of v_{max}. Erosional velocity can be estimated as:

$$v_{max} = \frac{100}{\sqrt{\rho}} = 100\sqrt{\frac{Z*R*T}{29*SG*P}}$$

Sonic Velocity (v_s) is the maximum possible velocity of a compressible fluid in a pipe.

$v_s = 68.1*[(C_p/C_v) P/\rho]^{0.5} = 68.1*[k*P/\rho]^{0.5}$

where, k = gas specific heat ratio =C_p/C_v

Mach Number (M_a) is the velocity of the gas divided by the sonic velocity in gas.
$M_a = v/ v_s$

4.0 Derivation of General flow Equation

Under Steady State conditions, the momentum equation can be written as:

Equ. 4.1

$$\rho^2 \mu\, du + \rho\, dP + \rho^2 g\, dH + f \frac{dx}{D}\frac{C^2}{2} = 0$$

where

$\rho^2 u\, du$ = the change in velocity or kinetic energy head,
$\rho\, dP$ = the change in pressure head,
$\rho^2 g\, dH$ = the change in elevation head, and
$f\, dx/D*C^2/2g$ = the friction head with f being the Darcy friction factor.

Setting $\rho u = \rho Q/A$ = constant C, the integration of the first term between velocities u_1 and u_2 becomes:

Equ. 4.2

$$\int_{u_1}^{u_2} \frac{C^2}{u} du = C^2 * ln\left(\frac{u_2}{u_1}\right)$$

Since $\rho = PM/zRT$, the integration of the second term between pressures P_1 & P_2 becomes:

Equ. 4.3

$$\int_{P_1}^{P_2} \rho\, dP = \int_{P_1}^{P_2} \frac{PM}{zRT}\, dP = \frac{M}{z_{avg}RT_{avg}} \int_{P_1}^{P_2} P\, dP = \frac{M}{z_{avg}RT_{avg}} * \frac{\left(P_2^2 - P_1^2\right)}{2}$$

Where
Equ. 4.4 $T_{avg} = (T_1 + T_2) / 2$

Since $\rho = PM/zRT$, the integration of the third term between elevations H_1 & H_2 becomes:

Equ. 4.5

$$\int_{H_1}^{H_2} \rho^2\, g\, dH = \int_{H_1}^{H_2} \left(\frac{PM}{zRT}\right)^2 g\, dH = \frac{g P_{avg}^2 M^2}{z_{avg}^2 R^2 T_{avg}^2}(H_2 - H_1)$$

Integration of the last term between point x1 and x2 along a pipe length L becomes:

Equ. 4.6

$$\int_{x_1}^{x_2} \frac{f * C^2}{2D_i} dx = fC^2 \frac{(x_2 - x_1)}{2D_i} = f\frac{L}{D_i}\frac{C^2}{2}$$

Bringing all integrated portions together, the momentum equation becomes:

Equ. 4.7

$$C^2 * ln\left(\frac{u_2}{u_1}\right) + \frac{M}{z_{avg}RT_{avg}} * \frac{(P_2^2 - P_1^2)}{2} + \frac{gP_{avg}^2 M^2}{z_{avg}^2 R^2 T_{avg}^2}(H_2 - H_1) + f\frac{L}{D_i}\frac{C^2}{2} = 0$$

The kinetic energy term is negligible in comparison to the other terms so the equation simplifies to:

Equ. 4.9

$$\frac{M}{z_{avg}RT_{avg}} * \frac{(P_2^2 - P_1^2)}{2} + \frac{gP_{avg}^2 M^2}{z_{avg}^2 R^2 T_{avg}^2}(H_2 - H_1) + f\frac{L}{D_i}\frac{C^2}{2} = 0$$

Since C = ρQ/A and ρ = PM/zR_uT at base conditions,

Equ. 4.10

$$C^2 = \frac{\rho_b^2 Q^2}{A^2} = \frac{16P_b^2 M^2 Q^2}{\pi^2 D_i^4 z_b^2 R_u^2 T_b^2}$$

Substituting Equ. 10 into the equation Equ. 9, and solving for Q^2,

$$Q^2 = \frac{\pi^2}{16}\frac{D^5}{L}\frac{2}{f}\frac{z_b^2 R_u^2}{M^2}\frac{T_b^2}{P_b^2}\left[\frac{M * (P_1^2 - P_2^2)}{2 * z_{avg}R_u T_{avg}} - \frac{gP_{avg}^2 M^2 * (H_2 - H_1)}{z_{avg}^2 R_u^2 T_{avg}^2}\right]$$

Solving for Q after substituting SG *M_{air} for molecular mass (M) and C_1 & C_2 for constants and simplifying, we get the **General Flow Equation (Equ. 4.11)**:

Equ. 4.11

$$Q = \frac{C_1}{\sqrt{f}} * z_b * D_i^{5/2} * \frac{T_b}{P_b} * \left[\frac{(P_1^2 - P_2^2) - C_2 * P_{avg}^2 * SG\frac{(H_2 - H_1)}{z_{avg} * T_{avg}}}{SG * L * z_{avg} * T_{avg}}\right]^{\frac{1}{2}} * \eta$$

Where,

$1/f^{1/2}$ = F_t = Von Karman transmission factor
η = efficiency factor with typical values between 0.8 and 1. Gas system modelers use the efficiency factor to adjust their model flow estimates based on actual measured conditions like metered flows and pressure readings. A value of 0.95 is commonly used.
L = Pipe Length [miles]
D_i = Pipe Inside Diameter [inches]
Q = Flow Rate [ft^3 / day]

$$C_1 = \left(\frac{\pi^2}{16} * \frac{R_u}{M_{air}}\right)^{\frac{1}{2}} * in^{\frac{5}{2}} * \frac{lb}{in^2} * \left(\frac{1}{mile}\right)^{\frac{1}{2}} * \left(\frac{1}{\text{oR}}\right)^{\frac{1}{2}} * \frac{oR}{lb/_{in^2}} * \left(\frac{12in}{1Ft}\right)^{\frac{1}{2}} * \frac{1Ft^3}{1728in^3} * \left(\frac{1\,Mile}{5280Ft}\right)^{\frac{1}{2}} * \frac{3600s}{1hr} * \frac{24hr}{1Day}$$

$$C_1 = \left(\frac{\pi^2}{16} * \frac{49762\,\frac{ft^2 * lb_m}{s^2 * lb_{mol} * oR}}{28.97\,\frac{lb_m}{lb_{mol}}}\right)^{\frac{1}{2}} * 2.3836 = 77.58\,\frac{ft^3 * in^{\frac{-5}{2}}}{day}\left(\frac{mile}{oR}\right)^{\frac{1}{2}}$$

$$C_2 = 2\text{g}\frac{M_{air}}{R_u} = \frac{2 * 32.2\,\frac{ft}{s^2} * 28.97\,\frac{lb_m}{lb_{mol}}}{49762\,\frac{ft^2 * lb_m}{s^2 * lb_{mol} * oR}} = 0.0375\,\frac{oR}{ft}$$

$P_{avg} = (2/3)[(P_1 + P_2 - (P_1*P_2)/(P_1 + P_2)]$ or $\frac{2}{3}\left[\frac{P_1^3 - P_2^3}{P_1^2 - P_2^2}\right]$

$T_{avg} = (T_1 + T_2) / 2$ [ºR] where ºR = ºF + 459.67

Replace L with L_E when estimating flow rates for two or more different pipe sizes in series (see **Figure 4.1**), where:

L_E = Equivalent Length = $\sum_{n=1}^{i} L_i \left[\frac{D_E}{D_i}\right]^{4.8539} \sim L_1 + L_2\left(\frac{d_1}{d_2}\right)^5 + L_3\left(\frac{d_1}{d_3}\right)^5 + \cdots$

D_E = Equivalent internal diameter (diameter you want to convert to)

D_i = Internal diameter of pipe section i with length L_i

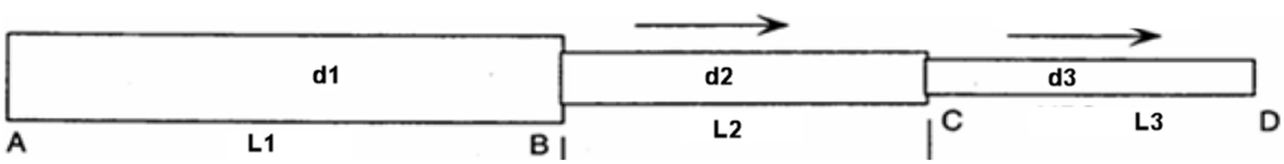


Figure 4.1 – Different Size Pipe in series

Equation 4.11 can be rewritten to solve for D and L where units are Q [cfs], L [Ft], P [psf], H [Ft], R_u $[\frac{ft^2*lb_f}{s^2 lb_{mol}*oR}]$

Equ. 4.12

$$D_i \text{ [inches]} = \frac{12}{\eta} * \left[\frac{P_b^2}{T_b^2} * \frac{16}{\pi^2} * \frac{f}{z_b^2} \frac{Q^2 * L}{\left[\frac{R_u(P_1^2 - P_2^2)}{SG * M_{air} * z_{avg} T_{avg}} - \frac{2gP_{avg}^2 * (H_2 - H_1)}{z_{avg}^2 T_{avg}^2}\right]}\right]^{1/5}$$

Equ. 4.13

$$L \text{ [Ft]} = \frac{\eta}{Q^2} \frac{\pi^2}{16} \frac{D^5}{f} \frac{z_b^2}{1} \frac{T_b^2}{P_b^2} \left[\frac{R_u(P_1^2 - P_2^2)}{SG * M_{air} z_{avg} T_{avg}} - \frac{2gP_{avg}^2 (H_2 - H_1)}{z_{avg}^2 T_{avg}^2}\right]$$

Solving for P_1 and P_2 is a little more difficult. The simple arithmetic equation for P_{avg} was substituted into Equ. 11 to simplify this solution. Inserting constants C4, C5, and C6 as defined below, we can begin to solve for P_1 and P_2.

Equ. 4.14

$$Q^2 = C_4\left[C_5(P_1^2 - P_2^2) - C_6 * \left[\frac{P_1 + P_2}{2}\right]^2\right]$$

Where

$$C_4 = \frac{\pi^2}{16} \frac{D^5}{L} \frac{1}{f} \frac{z_b^2 R_u}{M} \frac{T_b^2}{P_b^2}$$

$$C_5 = \frac{1}{z_{avg} T_{avg}}$$

$$C_6 = \frac{2g * M * (H_2 - H_1)}{z_{avg}^2 R_u T_{avg}^2}$$

$$C_7 = \frac{1}{4} C_6$$

Equ. 4.14 can be solved for P_1 or P_2 using the solution to the quadratic equation.

$$[C_4C_5P_1^2 - C_4C_5P_2^2 - C_4C_7P_1^2 - C_4C_7P_2^2 - 2C_4C_7P_1P_2] - Q^2 = 0$$

Solving for P_1, ($P_1 > P_2$)

$$P_1^2(C_4C_5 - C_4C_7) + P_1(-2C_4C_7P_2) + (-C_4C_5P_2^2 - C_4C_7P_2^2 - Q^2) = 0$$

Equ. 4.15

$$P_1 \text{ [psf]} = \frac{2C_4C_7P_2 \pm \sqrt{(2C_4C_7P_2)^2 - 4(C_4C_5 - C_4C_7)(-C_4C_5P_2^2 - C_4C_7P_2^2 - Q^2)}}{2(C_4C_5 - C_4C_7)}$$

Solving for P_2, ($P_2 < P_1$)

$$P_2^2(-C_4C_5 - C_4C_7) + P_2(-2C_4C_7P_1) + (C_4C_5P_1^2 - C_4C_7P_1^2 - Q^2) = 0$$

Equ. 4.16

$$P_2\ [\text{psf}] = \frac{2C_4C_7P_1 \pm \sqrt{(2C_4C_7P_1)^2 - 4(-C_4C_5 - C_4C_7)(C_4C_5P_1^2 - C_4C_7P_1^2 - Q^2)}}{2(-C_4C_5 - C_4C_7)}$$

Useful Conversions

1 slug = 32.17405 lb_m = 1 lb_f*s^2/ft

1 lb_m = 1 slug / 32.17405

1 lb_f = 1 slug*ft/s^2 = 32.17405 ft*lb_m/s^2

5.0 Estimating the Friction Factor, *f*

All gas flow equations are derived from the General Flow Equation. The differences in flow equations comes from the assumptions used to reduce the General Flow Equation and the estimate used for the Darcy friction factor, *f*. The iterative process using the Colebrook-White Equation typically give the best results for estimating the transmission factor (Ref. 2 & 3). This method takes in to account the relative roughness of the pipe and the Reynolds number. **Table 5.1** lists many methods for estimating the friction factor and the limits for their applicability. **Figure 5.1** graphically shows how the friction factor estimate varies over a range of Reynolds numbers for a few of the methods listed in Table 5.1. The gas calculator presents results based on using the following transmission factors:

- Colebrook-White (Modified)
- IGT Improved
- Chen
- Goudar-Sonnad (recommended by ref 3)
- Renouard (recommended by ref 10)

Figure 5.1 – Friction factor estimate variation over a range of Re values (Ref 10)

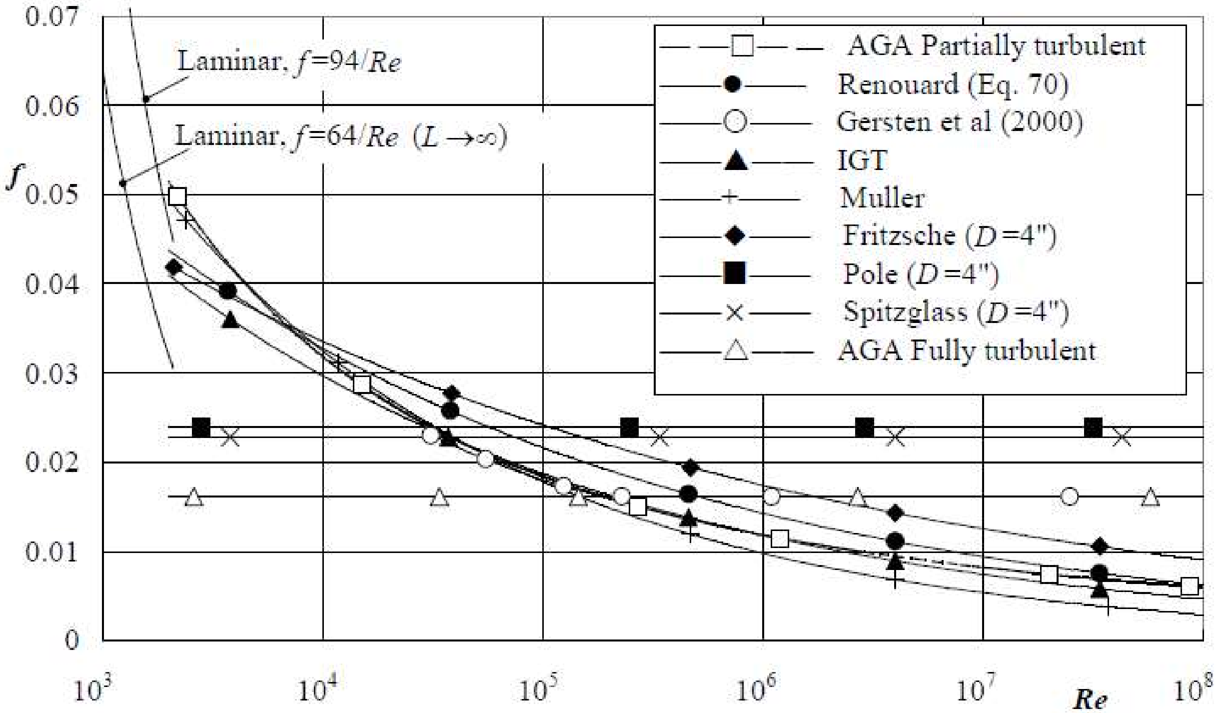


Table 5.1 - Approximations of the Colebrook-White Equation

Equation Name / Author	Ref.	Transmission Factor, F/2 = $1/f^{0.5}$	Applicable Reynolds Number Range	Applicable Pressure Range	Applicable Pipe Size Range	Applicable relative Roughness Range, ϵ = e/D	Applicable Flow Rate & Length	Year Developed
Colebrook-White	1,8	$-2\log_{10}\frac{2.51}{Re\sqrt{f}}$, turbulent flow in smooth pipes Initial estimate = $1.8\log_{10}\frac{Re}{6.9}$	4000 to 1E8					1933
Colebrook-White	2,3,4, 5,19	$-2\log_{10}\left(\frac{2.51}{Re\sqrt{f}}+\frac{\epsilon}{3.7}\right)$, turbulent flow	4000 to 1E8			0 - 0.05		
Colebrook-White	6	$1.74-2\log_{10}\left(2\in+\frac{18.7}{Re\sqrt{f}}\right)$, smooth pipes, transition to turbulent flow						
Colebrook-White (Modified)	5,8, 10	$-2\log_{10}[(\epsilon/3.7 + 2.825/(Re*f^{0.5})]$, where $-2\log_{10}[\epsilon/3.7]$, turbulent flow in rough pipes $-2Log_{10}[2.825/(Re*f^{0.5})]$ = turbulent flow in smooth pipes	4000 to 1E8			0 - 0.05	< 250 MMSCFD	1939
Approximations of the Colebrook-White Equation								
AGA Fully Turbulent	8, 10	$-2Log_{10}[\epsilon / 3.74]$	> 4000				< 250 MMSCFD	
AGA Partially Turbulent (Prandtl-von Karman)	10	$-2Log_{10}[(2.825/(Re*f^{0.5})]$	2000 to 4000					

AGA Partially Turbulent	7	$4Log_{10}[((Re*f^{0.5})] - 0.6$						
Achour	18	$-2*Log_{10}[\epsilon/3.7 + 4.5/Re*Log_{10}(Re/6.97)]$	>1E4			0 – 0.05		2002
Altshul	2,3, 18	$[0.11*(68/Re + \epsilon)^{0.25}]^{-0.5}$	4E3 – 1E7			0 to 0.01		1952
Avci & Karagoz	3	$0.3953[\ln(Re) - \ln(1+0.01\epsilon Re(1+10\epsilon^{0.5})]^{1.2}$						2009
Barr	3, 18	$-2*Log_{10}[\epsilon/3.7 + 4.518*Log_{10}(Re/7) / (Re*(1 + Re^{0.52}/(29\epsilon^{0.7}))]$	2300 to 1E8			0 to 0.05		1981
Blasius	1	$2.331(Re)^{1/10}$	>2E4					1913
Blasius	5	$1.7789*Re^{1/8}$	<1E5					
Brkic	18	$-2\log(10^{-0.4343\beta} + \epsilon/3.7)$ where $\beta = \ln(1+0.458Re)[1 - (\ln(1+0.458Re)/(2+\ln(1+0.458Re))]$		4000 to 1E8		0 – 0.05		2011
Buzzelli	3,18	$A - \left[\frac{A+2\log(B/Re)}{1+(2.18/B)}\right]$ where $A = \frac{0.744\ln(Re)-1.41}{1+1.32\sqrt{\epsilon}}$, $B = \frac{\epsilon}{3.7}Re + 2.51A$		2300 to 1E8		0 – 0.05		2008
Chen	2,3, 18	$-2*Log_{10}[\epsilon/3.7065 - 5.0452/Re * \log_{10}(\epsilon^{1.1096} / 2.8257 + 5.8506/Re^{0.8961})]$	4E3 to 4E8			1E-7 to .05		1979
Chen	4,6	$-4*Log_{10}[\epsilon/3.7065 - 5.0452/Re * \log_{10}(\epsilon^{1.1096} / 2.8257 + (7.149/Re)^{0.8961})]$	4E3 to 4E8					1979
Churchill	2,3, 18	$-2*\log_{10}[\epsilon/3.71 + (7 / Re)^{0.9}]$						1973

Eck	3,18	$-2\log_{10}[\epsilon/3.71 + 15 / Re]$						1973
Fang	2,3,18	$0.787\ln[0.234\epsilon^{1.1007} - 60.525/Re^{1.1105} + 56.291/Re^{1.0712}]$	3000 to 1E8			0 - 0.05		2011
Fritzsche	10	$3.3390(Re \cdot D)^{0.071}$						
Ghanbari–Farshad–Rieke	3, 18	$(-1.52\log[(\epsilon/7.21)^{1.042} + (2.731/Re)^{0.9152}])^{1.0845}$	2100 – 1E8			0 - 0.05		2011
Goudar-Sonnad	3,18, 19	$0.8686\ln[0.4587 Re / (C^{C/(C+1)})]$, where $C=0.124 Re \epsilon + \ln(0.4587 Re)$	4000 to 1E8			1E-6 – 0.05		2006
Hagen-Poiseuille	1,6	$(Re / 64)^{0.5}$	≤ 2100					1840
Haaland	2,3,5, 18	$-1.8\log_{10}[(\epsilon/3.7)^{1.11} + 6.9 / Re]$	4E3 to 1E8			1E-6 – 0.05		1983
IGT-Improved	4, 10	$2.3095 Re^{0.1}$	16,000 to 3E6	< 1 2-20psi 20-100psi	3" to 30" 1.5" to 20" 0.75" to 12"			1960's
Jain	3	$-2\log_{10}[\epsilon /3.715+ (6.943/Re)^{0.9}]$	5E3 to 1E8			4E-5 - 0.05		1976
Jain	2,6	$1.14 - 2\log_{10}[\epsilon + 21.25/Re^{0.9}]$	5E3 to 1E8			4E-5 - 0.05		1976
Manadilli	2,3,18	$-2\log_{10}[\epsilon/3.7 + 95/Re^{0.983} - 96.82/Re]$	4000 to 1E8	5200 to 1E8		0 - 0.5		1997
Moody	2,3,18	$[0.0055[1 + (2E4 \epsilon + 1E6/Re)^{1/3}]]^{-1/2}$	4000 to 5E8			0 - 0.01		1947
Morrison	1	$[[0.0076(3170/Re)^{0.165} / (1 + (3170/Re)^{7.0}] + 16/Re]^{-1/2}$	≤ 1E6					2013
Mueller - High	10	$1.675 Re^{0.13}$	2000 to 1.25E5	< 1 2-20psi 20-100psi	3/8" to 6" 3/8" to 2" 3/8" to 1.5"			
Nikuradse	4	$1.14 - 2\log_{10}[\epsilon]$	>4000					

Nikuradse	6	$1.74 - 2Log_{10}[2\epsilon]$						
Nikuradse	11	$3.476 - 4Log_{10}[\epsilon/3.7]$						
Oliphant	7	$1 + D^{0.5} / 30$						
Panhandle A	4,6, 10	$3.43 *Re^{0.0735}$	1.3E6 to 7.5E7	800 to 1500psi	12" to 60"			1940's
Panhandle B (Modified)	4,6, 10	$8.165*(Re)^{0.01961}$	4E6 to 40E6	> 1000psi	$D \geq 36"$			1956
Panhandle B	8	$16.7E*(Q*SG/D)^{0.01961}$						
Papaevangelou	3	$0.2479 - 9.47E\text{-}5(7\text{-}log(Re))^4 / (log(\epsilon/3.615 + 7.366/Re^{0.9142}))$						2010
Prandtl	1, 4	$2log_{10}(Re*f^{0.5}) - 0.8$	4000 to 1E6					1935
Prandtl - von Karman	10,18	$-2*log_{10}(2.825/(Re*f^{0.5})$, smooth pipe						
Prandtl - von Karman - Nikuradse	2	$2*log_{10}(Re*f^{0.5}) - 0.08$, smooth pipe $1.14 - 2*log10(\epsilon)$, rough pipe	4000 to 1E6					
Renouard - Low	10	$0.21*Re^{-0.2}$	< 4000					
Renouard - Medium	10	$2.4112*Re^{0.09}$	4000 to 4E6					1952
Renouard - High	10	$2.1822*Re^{0.1}$	> 4E6					
Round	2,3, 18	$-1.8*Log_{10}[Re /(0.135\epsilon Re + 6.5)]$	4000 to 4E8			0 - 0.05		1980
Shacham	3, 18	$-2*Log_{10}[\epsilon/3.7 - 5.02/Re*Log_{10}(\epsilon/3.7 + 14.5/Re)]$	4000 to 4E8			0 – 0.05		1980
Spitzglass -High	4	$[88.5 / (1 + 3.6/D + 0.03*D)]^{0.5}$		3psi to 100psi	$D \leq 10"$			1912
Spitzglass -Medium	10	$[88.5 / (1 + 0.09144/D + 1.1811*D)]^{0.5}$		1psi to 3psi				

Spitzglass -Low	4	$[88.5 / (1 + 3.6/D + 0.03*D)]^{0.5}$		< 1psi	D ≥ 16"			1912
Swamee, Jain	2,3,5, 18	$-2*\log_{10}(\epsilon/3.7 + 5.74 / Re^{0.9})$	5E3 to 1E7			1E-6 – 0.05		1976
Weymouth	4, 8, 10	$5.59*D^{1/6}$	> 4000	100 to 1000psi	D ≤12"		L < 20 miles	1912
White	10	$(1.02)^{-0.5} * (\log_{10}Re)^{1.25}$						1979
Wood	2,18	$[0.53\epsilon + 0.094\epsilon^{0.225} + 88\epsilon^{0.44} * Re^{(-1.62\epsilon\text{^}0.134)}]^{-0.5}$	4E3 – 5E7			0.00001 – 0.04		1966
Von Karman	8	$-2Log_{10}[\epsilon / 3.7]$ for rough pipe $2D_f*\log_{10}[Re/(1.412*F_t)]$ for smooth pipe						
von Karman and Prandtl	4	$2\log(Re*f^{1/2}) – 0.8$						
Zigrang, Sylvester	2,3,1 8	$-2*Log_{10}[\epsilon/3.7 - 5.02/Re * Log_{10}(\epsilon - 5.02/Re *Log_{10}(\epsilon/3.7 + 13/Re)]$	4E3 to 1E8			4E-5 – 0.05		1982

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